When Uncertainty Is the Target: Adversarial Attacks on Uncertainty-Aware Predictors

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Abstract

As machine learning becomes increasingly deployed in high-stakes scenarios, studying adversarial vulnerability of uncertainty-aware models has become a compelling necessity. In this paper, we focus on adversarial attacks that manipulate *predictive uncertainty* rather than model predictions. We focus on predictors that decompose uncertainty into aleatoric uncertainty (AU) and epistemic uncertainty (EU) and formalize *uncertainty vulnerability* as the maximum increase in AU achievable under a stealth constraint where the EU is kept below a threshold. This framing yields three complementary findings. First, we theoretically show that knowledge of the baseline EU can enlarge the attacker's feasible stealth region and increase the maximal achievable increase in AU. Second, a first-order analysis reveals that vulnerability is governed by the component of the AU gradient orthogonal to EU, enabling stealthy manipulation. Third, contrary to standard margin-based intuition, vulnerability peaks at moderate margins. Experiments across MNIST, CIFAR-10, and CIFAR-100 with standard architectures show that the proposed attack achieves non-trivial aleatoric damage while maintaining zero epistemic-stealth violation (e.g., mean AU increase of 0.0286, 0.0853, and 0.1568, respectively), whereas unconstrained uncertainty attacks violate the stealth constraint on over 90% of samples. Experiments indicate that the largest damage typically occurs in an intermediate uncertainty regime rather than at the extremes. Furthermore, consistent with our theoretical analysis, feasible attackability collapses under increasing epistemic uncertainty (e.g., mean attackability decreases from 0.0363 to 0). Overall, these results intimately connect uncertainty decomposition to adversarial geometry.

1 Introduction

Predictive uncertainty is increasingly used in high-stakes systems to properly inform decision-making. Real-world failures such as biased recognition systems [Guynn, 2015, AlgorithmWatch, 2020, Medina, 2023] and autonomous driving incidents [National Highway Traffic Safety Administration, 2017] show that unreliable confidence or uncertainty estimates can be as dangerous as incorrect predictions themselves. This has motivated a large body of work on uncertainty quantification (UQ) for deep neural networks (DNNs), including Bayesian [Kendall and Gal, 2017, Depeweg et al., 2018], ensemble-based [Lakshminarayanan et al., 2017], imprecise probability [Caprio et al., 2024, 2025, Amini et al., 2020] and distributional [Malinin and Gales, 2018] approaches.

Predictive uncertainty is distinguished into aleatoric uncertainty (AU) and epistemic uncertainty (EU). AU captures ambiguity intrinsic to the input (thus irreducible), such as class overlap or label noise, whereas EU reflects model uncertainty arising from insufficient knowledge or weak support in the training distribution – and it is thus reducible [Kendall and Gal, 2017, Depeweg et al., 2018]. While this decomposition has become central to modern uncertainty quantification, *we argue that an adversary could only need to manipulate the uncertainty associated with the prediction to inflict damage, instead of attempting to change the predicted labels*.

While adversarial machine learning has extensively been studied attacks on model predictions (e.g., evasion attacks), *attacks to model uncertainty are largely underexplored*. To our knowledge, no prior work studies *attacks that increase AU while remaining stealthy with respect to EU*. While existing work on calibration shows that predictive confidence can be made unreliable under adversarial perturbations [Guo et al., 2017, Kull et al., 2019, stu], they largely treat uncertainty as a single scalar quantity (e.g., confidence or total predictive entropy). In contrast, a more fundamental setting – attacking AU while keeping the EU unchanged – remains open.

Figure 1 illustrates the difference between classic evasion attacks and uncertainty-targeted attacks. In the former, the adversary attempts to make the model confidently wrong: for example, a cat image is perturbed so that the predictor assigns high probability to "dog." *Conversely, an attack against an uncertainty-aware predictor can be subtler*. The adversary may avoid forcing a confident wrong answer and make the prediction appear genuinely ambiguous, e.g., by spreading probability mass between "cat" and "dog," while keeping the system's warning signal low. As such, the downstream system may then treat the input as an intrinsically difficult case instead of as a manipulation, since from the defender's perspective, increased AU is indistinguishable from genuine data ambiguity.

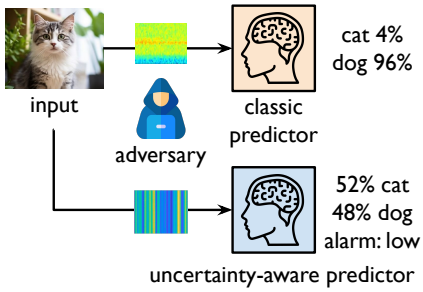

Figure 1: Classical evasion attacks aim to make a predictor confidently wrong; uncertainty-targeted attacks make the model appear legitimately unsure, while the warning signal remains low.

In this work, we formalize and analyze adversarial attacks that maximize AU while keeping EU within a prescribed stealth budget. What makes this problem challenging is that the attacker must increase AU without exceeding a given perturbation budget for the EU. These two quantities are coupled; as such, perturbations that are typically harmful for prediction might not always be successful. This leads to the following intriguing question: *what helps the attacker successfully increase AU without increasing EU beyond a certain level?* One of our key findings is that this form of uncertainty manipulation is not uniformly achievable across samples. Conversely, vulnerability tends to peak at neither very low nor very high baseline uncertainty but at intermediate uncertainty regime. We hypothesize that these samples lie in transition regions of the classifier (i.e., where class ambiguity is appreciable but epistemic disagreement remains sufficiently constrained) and that this region is closely related to traditional notions of adversarial vulnerability such as small margin and high local sensitivity. *This perspective leads to a new view of adversarial robustness: by characterizing which samples are most vulnerable to uncertainty manipulation, we connect uncertainty decomposition with the classifier's geometry and identify the conditions under which uncertainty-aware predictors are most vulnerable*.

Summary of Novel Contributions

• We formulate sample-wise uncertainty vulnerability as a constrained optimization problem – maximize the increase in AU while remaining stealthy in EU. This isolates a class of attacks that alters the reliability signal without necessarily forcing immediate label change.

• We show that the attacker's feasible damage budget depends on how baseline EU is monitored or disclosed. In particular, we derive conditions under which knowledge of the baseline epistemic level enlarges the attacker's opportunity set, while imperfect knowledge shrinks it. This is corroborated empirically by the collapse of maximum feasible damage (e.g., mean Γ decreases from 0.0363 to 0 as epistemic uncertainty radius increases). We derive a first-order characterization of local vulnerability, and show that vulnerability is amplified by aleatoric changes that are orthogonal to epistemic changes and is often largest at moderate margins.

• We validate these predictions empirically across MNIST, CIFAR-10, and CIFAR-100 using multiple ensemble architectures. Across settings, the maximal feasible aleatoric damage is typically non-monotone in baseline uncertainty and is concentrated in intermediate uncertainty regimes The constrained attack achieves significant AU damage (e.g., mean ΔAU of 0.0286 on MNIST, 0.0853 on CIFAR-10, and 0.1568 on CIFAR-100), while maintaining zero EU violations, as opposed to unconstrained attacks that violate EU constraints 90% of the time.

2 Related Work

Adversarial vulnerability, margin, and local sensitivity. Literature has shown that samples with smaller effective margin or that lie closer to a decision boundary are generally more susceptible to adversarial perturbations. This view has motivated margin-based robustness methods such as Lipschitz-margin training [Tsuzuku et al., 2018] and adversarial margin maximization [Yan et al., 2019]. More broadly, it is consistent with the intuition that a larger perturbation is required to alter the prediction of a sample that lies deeper inside a class region. Related geometric analyses directly study surrounding decision boundaries and show that boundary distances and adjacent-class structure around a sample help characterize adversarial susceptibility [He et al., 2018]. A complementary line of work relates adversarial vulnerability to local sensitivity, often quantified through input gradients or local Lipschitz behavior. [Goodfellow et al., 2015] argued that adversarial examples arise in part from the locally linear behavior of modern networks, while [Simon-Gabriel et al., 2019] showed that first-order adversarial vulnerability grows with gradients of the training objective. Similarly, [Finlay and Oberman, 2021] derived per-sample robustness bounds based on local gradient information and proposed scalable input-gradient regularization as a robustness mechanism. More recent training methods explicitly reweight samples by geometric vulnerability, emphasizing points closer to the boundary during adversarial training [Zhang et al., 2021]. Our work instead studies where the uncertainty decomposition is easiest to manipulate, which is a fundamentally different problem.

Attacks on confidence, calibration, and uncertainty. Existing work has shown that neural networks are often miscalibrated [Guo et al., 2017, Kull et al., 2019] and that calibration and predictive uncertainty can substantially degrade under adversarial perturbations [stu]. These findings imply that uncertainty-based decision systems may fail due to corruption of their reliability. As such, a growing literature now studies direct attacks on uncertainty estimates. Early work showed that confidence scores can be manipulated without necessarily changing the predicted labels [Galil and El-Yaniv, 2021, Tuna et al., 2023]. Other work attacked EU estimates derived from Bayesian approximations or MC-dropout models, showing that uncertainty itself can be exploited as an attack objective [Tuna et al., 2021]. More recently, adversarial perturbations have been shown to severely distort system-level calibration metrics such as ECE and Brier score while leaving accuracy nearly unchanged [Emde et al., 2025]. Complementarily, robustly trained classifiers have been shown to exhibit smaller "uncertainty spans" under perturbation, indicating that adversarial robustness can partially transfer to uncertainty robustness [Ledda et al., 2026]. Instead, our work asks how an attacker can manipulate an AU signal while remaining stealthy with respect to EU monitoring.

Summary of Novelty: This work studies a more selective form of uncertainty attack than prior attacks, as we ask whether an adversary can increase AU while keeping the monitored EU nearly unchanged. This leads to a constrained, sample-wise vulnerability measure: the maximum apparent ambiguity that can be induced without triggering an epistemic warning. The resulting analysis connects uncertainty manipulation to local gradient geometry and classifier margin, showing that vulnerability is controlled not only by how easily AU can be increased, but by whether it can be increased in directions that leave EU stable. This explains why attackability often peaks in an intermediate uncertainty regime instead of at the extremes.

3 Problem Setup

Let $(\mathcal{X}, d_{\mathcal{X}})$ and $(\mathcal{Y}, d_{\mathcal{Y}})$ be our input and output metric spaces, assumed to be *separable*. We use a predictive credal approach [Caprio et al., 2024] so that, given a training set $D = \{(x_i, y_i)\}_{i=1}^n$ and a new test input $x_{n+1} \equiv x$, we obtain a predictive set $\mathcal{P}_x$ on the y's, i.e. $\mathcal{P}_x \subseteq \Delta_{\mathcal{Y}}$ convex and weak*-closed, where $\Delta_{\mathcal{Y}}$ is the space of (countably additive) probability measures on $\mathcal{Y}$. **Assumption (Attacker Perfect Knowledge).** The attacker knows $\mathcal{P}_x$ and seeks a

perturbation $\tilde{x}$ solving

$$\begin{aligned} \inf_{\tilde{x}\in\mathcal{X}} \quad & d_{\mathcal{X}}(\tilde{x}, x) \\ \text{s.t.} \quad & d_H(\mathcal{P}_x, \mathcal{P}_{\tilde{x}}) \leq \delta \\ & AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \geq \gamma, \end{aligned} \tag{1}$$

where d_H is the Hausdorff distance on $\Delta_{\mathcal{Y}}$ and $AU(\cdot)$ denotes AU (e.g., lower entropy or lower prevision). Problem (1) enforces stealth via proximity of the full predictive object. In practice, defenders often monitor only a scalar $EU(\mathcal{P}_x)$, leading to the relaxed formulation

$$\begin{aligned} \inf_{\tilde{x}\in\mathcal{X}} \quad & d_{\mathcal{X}}(\tilde{x}, x) \\ \text{s.t.} \quad & |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta \\ & AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \geq \gamma, \end{aligned} \tag{2}$$

where $EU(\cdot)$ denotes EU (e.g., entropy gap or upper–lower prevision difference). This constraint is strictly weaker than (1), since similar EU values do not imply proximity in d_H.

The defender's notion of stealth depends on the monitored predictive object: **Full monitoring:** $d_H(\mathcal{P}_x, \mathcal{P}_{\tilde{x}}) \leq \delta$. **Scalar EU monitoring:** $|EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta$. **Uncertain baseline EU:** $EU(\mathcal{P}_x)$ is not exactly known to the attacker and lies within an uncertainty set. We focus on scalar EU monitoring (second regime) as this is more practical for deployed systems, with the third treated as a special case capturing partial epistemic disclosure. Throughout this work, we view uncertainty manipulation through three equivalent lenses. The primary object is the predictive credal set $\mathcal{P}_x$. Associated uncertainty functionals AU(·) and EU(·) give a scalar view of the predictive credal sets. For local analysis, we further write $a(x) := AU(\mathcal{P}_x)$ and $e(x) := EU(\mathcal{P}_x)$ making the connection to input space explicit. The set-level, scalar, and local view will be used interchangeably.

4 Baseline Epistemic Disclosure and Vulnerability

We focus on Problem (2), as it is more realistic that the defender does not (or cannot) monitor the full predictive object $\mathcal{P}_x$ and instead watches one or a few scalar diagnostics. We define the global and local maximum feasible damage as the following.

Definition 4.1 (Global Maximal Feasible Damage Budget)**.** Fix $\delta > 0$ and let $\mathfrak{P} := \{\mathcal{P}_x : x \in \mathcal{X}\}$ be the family of predictive credal sets. Let $EU, AU : \mathfrak{P} \to \overline{\mathbb{R}} := \mathbb{R} \cup \{\infty\}$ be functionals such that $EU(\mathcal{P}) \geq 0$ for all $\mathcal{P} \in \mathfrak{P}$. For $x \in \mathcal{X}$, define

$$\Gamma(x, \delta) := \sup \Big\{ \gamma \in \mathbb{R} : \exists \tilde{x} \in \mathcal{X} \text{ s.t. } |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta \text{ and } AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \geq \gamma \Big\}.$$

Equivalently,

$$\Gamma(x, \delta) = \sup_{\tilde{x}\in\mathcal{X}: |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \Big] \in \overline{\mathbb{R}}.$$

Definition 4.1 formalizes sample-wise vulnerability or attackability under epistemic-stealth monitoring with $\Gamma(x, \delta)$. This denotes the largest increase in AU achievable for a sample while keeping the EU within a δ-band around its baseline value. In practice, it is not feasible to estimate the global maximum feasible damage budget as there would be no bound on the added perturbation. In practice, we estimate a local version of it defined as follows:

Definition 4.2 (Local Maximal Feasible Damage Budget)**.** Fix $\epsilon > 0$ and $\delta > 0$, and let $\mathfrak{P} := \{\mathcal{P}_x : x \in \mathcal{X}\}$ be the family of predictive credal sets. Let $EU, AU : \mathfrak{P} \to \overline{\mathbb{R}} := \mathbb{R} \cup \{\infty\}$ be functionals such that $EU(\mathcal{P}) \geq 0$ for all $\mathcal{P} \in \mathfrak{P}$. For $x \in \mathcal{X}$, define

$$\Gamma_\epsilon(x, \delta) := \sup_{\tilde{x}\in\mathcal{X}:\ \|\tilde{x}-x\|_\infty \leq \epsilon,\ |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \Big].$$

Thus $\Gamma_\epsilon(x, \delta)$ measures the largest achievable increase in AU inside an ℓ_∞ perturbation ball of radius ϵ, while remaining stealthy with respect to EU up to tolerance δ. In practice, we estimate $\Gamma_\epsilon(x, \delta)$ via PGD-based constrained optimization.

In the subsequent sections, unless otherwise stated, any Γ or Γ_ϵ refer to $\Gamma_\epsilon(x, \delta)$ introduced in the definition 4.2. The following theorem gives an equivalence characterization for the relation between EU and AU. Specifically, it shows that $EU(\mathcal{P}_x) = 0$ yields a smaller maximum damage budget than $EU(\mathcal{P}_x) > 0$.

Theorem 4.3 (Iff Characterization of the relation between EU and AU)**.** *Fix $\delta > 0$ and assume $EU, AU : \mathfrak{P} \to \overline{\mathbb{R}}$ with $EU(\mathcal{P}) \geq 0$ for all $\mathcal{P} \in \mathfrak{P}$. Let $x_0, x_1 \in \mathcal{X}$ satisfy $EU(\mathcal{P}_{x_0}) = 0$, and $EU(\mathcal{P}_{x_1}) =: e > 0$. We define the following two (nonempty) envelope values,*

$$A_0(\delta) := \sup_{\tilde{x} \in \mathcal{X}: EU(\mathcal{P}_{\tilde{x}}) \leq \delta} AU(\mathcal{P}_{\tilde{x}}), \qquad A_1(e, \delta) := \sup_{\tilde{x} \in \mathcal{X}: \max(0, e-\delta) \leq EU(\mathcal{P}_{\tilde{x}}) \leq e+\delta} AU(\mathcal{P}_{\tilde{x}}).$$

Then the following statements are equivalent:

(i) $\Gamma(x_0, \delta) < \Gamma(x_1, \delta)$.

(ii) The strict envelope inequality holds:

$$A_1(e, \delta) - A_0(\delta) > AU(\mathcal{P}_{x_1}) - AU(\mathcal{P}_{x_0}). \tag{3}$$

(iii) There exists $\hat{x} \in \mathcal{X}$ such that $|EU(\mathcal{P}_{\hat{x}}) - e| \leq \delta$ and

$$AU(\mathcal{P}_{\hat{x}}) - AU(\mathcal{P}_{x_1}) > \sup_{\tilde{x} \in \mathcal{X}: EU(\mathcal{P}_{\tilde{x}}) \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_{x_0}) \Big]. \tag{4}$$

In particular, Equation (3) *gives a necessary and sufficient condition for the maximum feasible damage budget (i.e., the largest γ for which the constraints in Equation* (2) *are feasible) to be strictly smaller when $EU(\mathcal{P}_x) = 0$ than when $EU(\mathcal{P}_x) = e > 0$.*

Key Finding: *Theorem 4.3 shows that disclosing the baseline EU can strictly enlarge the attacker's stealth-feasible opportunity set. When the baseline value $e = \mathrm{EU}(\mathcal{P}_x)$ is positive and known, the attacker can center the stealth constraint around e, rather than being restricted to the low-epistemic region induced by $\mathrm{EU}(\mathcal{P}_x) = 0$. This may expose perturbed inputs whose predictive credal sets achieve substantially larger AU. The key quantity is the opportunity gap $A_1(e, \delta) - A_0(\delta)$: whenever this gap exceeds the corresponding difference in baseline aleatoric uncertainty, the attacker can achieve a strictly larger feasible increase in AU under exact epistemic disclosure.*

Problem (2) presumes that the attacker knows the baseline EU $e := EU(\mathcal{P}_x)$ and thus can enforce the pointwise stealth constraint $|EU(\mathcal{P}_{\tilde{x}}) - e| \leq \delta$. If instead e is not computed or not disclosed, the attacker may only know that $e \in \mathcal{E}_x$ for some uncertainty set $\mathcal{E}_x \subseteq \mathbb{R}_+$. A conservative attacker who wishes to guarantee stealth for all admissible baselines must satisfy the *robust stealth* condition

$$\sup_{e \in \mathcal{E}_x} |EU(\mathcal{P}_{\tilde{x}}) - e| \leq \delta. \tag{5}$$

Let $u := EU(\mathcal{P}_{\tilde{x}})$. Writing $\mathcal{E}_x = [\underline{e}_x, \overline{e}_x]$ (with $\underline{e}_x := \inf \mathcal{E}_x$ and $\overline{e}_x := \sup \mathcal{E}_x$), (5) is equivalent to

$$u \in [\underline{e}_x - \delta, \underline{e}_x + \delta] \cap [\overline{e}_x - \delta, \overline{e}_x + \delta] = \big[\overline{e}_x - \delta, \underline{e}_x + \delta\big]. \tag{6}$$

Hence robust stealth is feasible if and only if $\overline{e}_x - \underline{e}_x \leq 2\delta$, and, whenever feasible, it shrinks the admissible epistemic region compared to the perfect-knowledge band $[e - \delta, e + \delta]$. In particular, as $\mathrm{diam}(\mathcal{E}_x) := \overline{e}_x - \underline{e}_x$ grows, the robust stealth region (6) contracts and may even become empty. Thus, imperfect epistemic knowledge can reduce the attackability by shrinking or even eliminating the feasible stealthy region. We discuss this further in Appendix F.

5 First-order condition for local attackability

While our analysis of the Problem 2 provides an equivalence characterization, it does not indicate when the conditions may hold. In this section, we perform first order analysis to get conditions when a sample can be successfully attacked and subsequently, Theorem 4.3 can hold.

Proposition 5.1 (First-Order vulnerability Condition)**.** *Let $a(x) := AU(\mathcal{P}_x)$ and $e(x) := EU(\mathcal{P}_x)$ be continuously differentiable in a neighborhood of x. Define*

$$\Gamma_{\mathrm{lin}}(x, \varepsilon) := \sup_{\|r\|_2 \leq \varepsilon,\ \nabla e(x)^\top r = 0} \nabla a(x)^\top r.$$

If $\nabla e(x) \neq 0$*, then*

$$\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon \left\| \Pi_{\nabla e(x)^{\perp}} \nabla a(x) \right\|_2 ,$$

where Π_m *denotes orthogonal projection onto the subspace* m*. If* $\nabla e(x) = 0$*, then*

$$\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon \|\nabla a(x)\|_2 .$$

Proposition 5.1 identifies the local mechanism behind stealthy uncertainty attacks. The key quantity is not the full aleatoric gradient, but the component that remains after removing the direction aligned with epistemic change. When $\nabla AU(x)$ points mostly along $\nabla EU(x)$, increasing AU inevitably changes EU as well, making stealth difficult. When a large orthogonal component exists, the attacker can increase AU while remaining locally invisible to EU-based monitoring.

Proposition 5.2 (Moderate-Margin Peak of Local Aleatoric Sensitivity)**.** *Let* $m(x) = z_{(1)}(x) - z_{(2)}(x) > 0$ *denote the top-two logit margin at a correctly classified sample* x*, and define* $a(x) := h(\sigma(m(x)))$*, where* $\sigma(t) = 1/(1 + e^{-t})$ *and* $h(p) = -p \log p - (1 - p) \log(1 - p)$*. Assume* m *is differentiable near* x*. Then*

$$\nabla a(x) = -\phi(m(x))\, \nabla m(x), \qquad \phi(t) := \sigma(t)(1 - \sigma(t))\, t,$$

where $\phi(0) = 0$*,* $\phi(t) \to 0$ *as* $t \to \infty$*, and* ϕ *attains its unique maximum at the positive solution* $t^{\star} \approx 1.5434$ *of* $t \tanh(t/2) = 1$*.*

Hence $\|\nabla a(x)\|_2 = \phi(m(x)) \|\nabla m(x)\|_2$ *is maximized, for fixed* $\|\nabla m(x)\|_2$*, at moderate margin values. If in addition* $e(x) := EU(\mathcal{P}_x)$ *is differentiable, then under exact first-order epistemic stealth,*

$$\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon\, \phi(m(x)) \, \left\| \Pi_{\nabla e(x)^{\perp}} \nabla m(x) \right\|_2 .$$

Key Finding: *Proposition 5.2 clarifies why vulnerability is not expected to peak at either vanishing or very large margins. The margin-modulation factor* $\phi(m)$ *is small at both extremes and attains its maximum at an intermediate margin; therefore, for a fixed margin-gradient norm, local aleatoric sensitivity is strongest for moderate-margin samples. Together with Proposition 5.1, this gives a sharper local prediction: the most attackable inputs are those whose margins lie in this intermediate regime and whose margin gradients have a large component orthogonal to the epistemic-gradient direction. In such cases, an adversary can increase apparent ambiguity while remaining locally stealthy with respect to epistemic monitoring.*

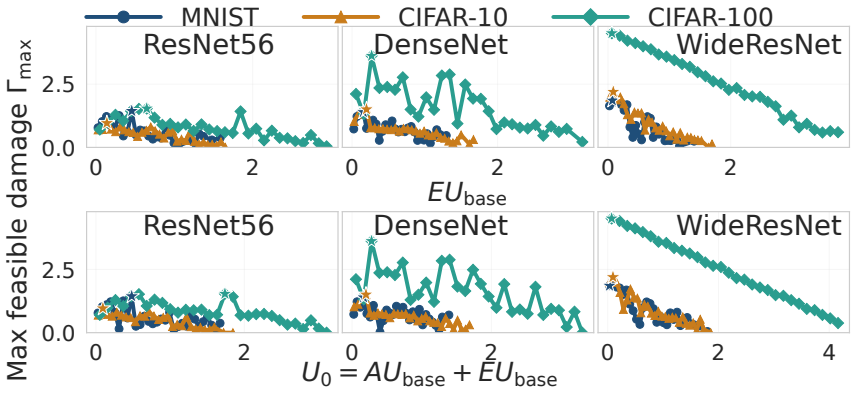

Figure 2: Upper envelope of feasible aleatoric damage across baseline uncertainty budgets. Columns correspond to architectures. The top row bins by baseline EU EU_{base}, and the bottom row bins by total baseline uncertainty $U_0 = AU_{\text{base}} + EU_{\text{base}}$. Across settings, the envelope bends and eventually declines rather than increasing monotonically, consistent with the theoretical prediction that the largest feasible damage occurs in an intermediate regime.

6 Experimental Setup and Results

In this section we empirically validate the theoretical predictions of Theorem 4.3, and Propositions 5.1-5.2. We provide more details of our experimental setup in Appendix C.

Datasets: MNIST [Deng, 2012], CIFAR-10, and CIFAR-100 [Krizhevsky and Hinton, 2009]. For these benchmarks, we train ensemble models using the CredRO [Wang et al., 2026] procedure.

Architectures: ResNet56 [He et al., 2016], WideResNet [Zagoruyko and Komodakis, 2016], and DenseNet [Huang et al., 2017] trained on each dataset mentioned above. Training details and hyperparameters are given in Appendix C.

Non-monotonic Maximum Feasible Damage Budget. We first study the maximum local feasible damage to identify which samples are more vulnerable. The relevant empirical object is therefore the upper envelope of attainable vulnerability Γ_ϵ. Figure 2 reports this envelope as a function of baseline EU (EU_{base}) and total baseline uncertainty ($U_0 = AU_{\text{base}} + EU_{\text{base}}$) across datasets and architectures. The non-monotone pattern is clearest when conditioning on EU_{base}. On MNIST, the envelope peaks in low-to-intermediate uncertainty bins for all three architectures and then declines. The same qualitative behavior persists on CIFAR-10 – peak vulnerability is attained away from the highest-EU regime, while the largest-EU bins exhibit substantially smaller $\Gamma_\epsilon(x, \delta)$. On CIFAR-100, the precise peak location becomes more architecture-dependent, but the envelope again bends and declines rather than rising monotonically. Thus, across all nine dataset–architecture pairs, the most attackable samples are generally not the most epistemically uncertain ones.

The same conclusion holds when uncertainty is measured through the total budget U_0. For CIFAR-10, all three architectures attain maximal envelope values at intermediate U_0. On MNIST and CIFAR-100, some models peak at low total uncertainty while others peak at intermediate uncertainty, but the envelope remains consistently non-monotone. This leads to the conclusion that maximal feasible aleatoric damage does not peak at high uncertainty region, rather at some intermediate uncertainty.

Table 1: Comparison of Spearman's rank correlation (ρ) between raw and projected gradient and maximum feasible aleatoric damage.

Dataset	Arch	ρ(raw)	ρ(projected)	Gain
MNIST	ResNet56	0.7836	0.7901	**0.0065**
	WideResNet	0.6058	0.6601	**0.0543**
	DenseNet	0.6685	0.7042	**0.0357**
CIFAR-10	ResNet56	0.1247	0.1250	**0.0003**
	WideResNet	0.3011	0.3011	**0.0001**
	DenseNet	0.2130	0.2134	**0.0004**
CIFAR-100	ResNet56	-0.3298	-0.3256	**0.0042**
	WideResNet	0.0750	0.0830	**0.0080**
	DenseNet	-0.2030	-0.1964	**0.0066**

First-Order Attack Criterion. We next test Proposition 5.1, which predicts that local vulnerability is governed not by the full aleatoric gradient $\|\nabla AU\|_2$, but by the component orthogonal to epistemic change,$\|\Pi_{\nabla EU^\perp} \nabla AU\|_2$. Intuitively, only directions that increase AU while leaving EU unchanged remain feasible under stealth constraints.

Table 1 summarizes the correlations between realized vulnerability Γ and these first-order quantities. On MNIST, the projected gradient consistently exhibits stronger monotone association with Γ across all architectures than raw gradient. For example, on ResNet56 the Spearman correlation increases from 0.784 for $\|\nabla AU\|_2$ to 0.790 after projection, with larger gains on WideResNet (0.606to0.660) and DenseNet (0.669to0.704). These results closely match the first-order geometric mechanism of Proposition 5.1. On CIFAR-10, the same trend is weaker but still partially visible: the projected statistic is marginally stronger on DenseNet and nearly unchanged on the remaining models. On CIFAR-100, correlations become mixed and occasionally negative, indicating that first-order local geometry alone is insufficient in harder recognition settings. This is consistent with a more complex attack landscape in which curvature, optimization effects, and higher-order interactions become increasingly important.

Table 2: Relation between classifier margin and maximum feasible attack damage.

Dataset	Arch	$\rho(\Gamma, \text{margin})$	Interpretation
MNIST	ResNet56	-0.6218	Strong negative
	WideResNet	-0.4784	Strong negative
	DenseNet	-0.5914	Strong negative
CIFAR-10	ResNet56	-0.1743	Moderate negative
	WideResNet	-0.3199	Moderate negative
	DenseNet	-0.2281	Moderate negative
CIFAR-100	ResNet56	0.3139	Mixed
	WideResNet	-0.1031	Mixed
	DenseNet	0.1745	Mixed

Overall, the evidence supports a clear hierarchy - first-order projected gradients are strongly predictive on simpler datasets, partially predictive on moderate ones, and less decisive on harder regimes. Thus, Proposition 5.1 is best viewed as a local mechanism that explains vulnerability where the uncertainty landscape is sufficiently smooth.

Relation Between Classifier Margin and vulnerability. We next examine whether uncertainty vulnerability is related to classical margin geometry. We use a classifier-level top-two margin surrogate. For each ensemble member

with classifier weights $\{w_i\}$ and predicted label $\hat{y}$, we compute $m_{\text{cls}}(x) = \frac{z_{\hat{y}}(x) - z_{(2)}(x)}{\|w_{\hat{y}} - w_{(2)}\|_2}$, and average across ensemble members. This quantity is not the exact input-space boundary distance, but it is the natural representation-space analogue. Table 2 shows a clear pattern on MNIST and CIFAR-10. On MNIST, vulnerability Γ is strongly negatively associated with classifier margin across all architectures, with Spearman correlations of -0.622 (ResNet56), -0.478 (WideResNet), and -0.591 (DenseNet). The same trend persists on CIFAR-10, though more weakly: -0.174, -0.320, and -0.228, respectively. Thus, on simpler and moderately difficult datasets, samples farther from the decision boundary are generally harder to manipulate under uncertainty-stealth constraints. We also test the more specific prediction of Proposition 5.2 by replacing raw margin with the modulation factor $\phi(m) = m\,\sigma(m)(1 - \sigma(m))$. Figure 3 visualizes this effect for the representative ResNet56 run on CIFAR100. We provide the figures for MNIST and CIFAR10 with resnet56 architecture in the Appendix. We notice that this quantity has a noticeably more stable positive relation with vulnerability. We can clearly observe the trend that the attackability or vulnerability peaks near the center(at a value of 5) of the top-two logit margin distribution. While the margin modulation does not follow this trend exactly due to the nonlinear effect of the sigmoid, it still peaks at an intermediate value and starts sloping downwards after that. This corroborates our analysis in Proposition 5.2 that the vulnerability is not necessarily at the extreme margin values but rather at an intermediate value.

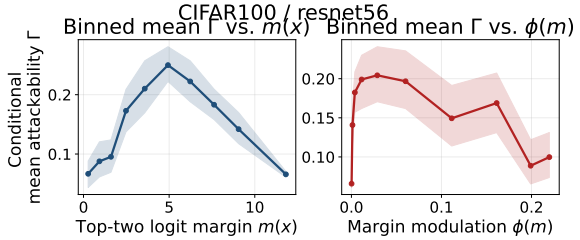

Figure 3: Binned conditional mean attackability $\Gamma_{\epsilon,\delta}(x)$ versus top-two logit margin $m(x)$ (left) and versus the modulation factor $\phi(m(x))$ (right) for CIFAR-100 / ResNet56. Shaded bands show 95% bootstrap confidence intervals. In this harder setting, the raw-margin relation is non-monotone with a peak at intermediate margins, and the $\phi(m)$ panel similarly concentrates larger vulnerability away from both extremes.

Disclosure Ablation. We test the claim that uncertainty about the baseline epistemic level can shrink the feasible stealth region by running a disclosure-ablation experiment on MNIST ResNet56 over the first 1000 test samples. We compare three attackers: a perfect-knowledge attacker that enforces stealth relative to the exact baseline e, a robust attacker that only knows an interval $[e - r, e + r]$, and a midpoint attacker whose perturbation is optimized using the midpoint but evaluated against the robust interval. Figure 4 summarizes how performance degrades as the uncertainty radius r grows. At $r = 0$, all three attackers coincide. As r increases, the robust attacker's feasible damage decreases monotonically: its mean Γ drops from 0.0363 at $r = 0$ to 0.0341 at $r = 0.01$, 0.0288 at $r = 0.02$, 0.0142 at $r = 0.05$, and finally 0 at $r = 0.10$. The midpoint attacker fails even faster under robust evaluation: its feasible rate falls from 1.000 at $r = 0$ to 0.948, 0.937, 0.483, and then 0 across the same radii. The mean stealth-band width contracts from 0.0550 at $r = 0$ to 0.0514, 0.0504, 0.0450, and finally 0. Once the robust band collapses at the largest radius, robust attackability vanishes entirely. This provides direct empirical support for the claim that non-disclosure of baseline EU can reduce attackability by shrinking or eliminating the stealth-feasible action set.

Baseline Comparison to Other Attack Objective. In order to understand how attacking the decomposed uncertainty is different than attacking alternate uncertainty objectives, we compare with four baselines: AU-only maximization, entropy maximization, PGD for evasion, and random ℓ_∞. Figure 5 summarizes the central tradeoff: mean AU increase versus the rate at which the epistemic-stealth constraint is violated.

From Figure 5, it is readily observable that attacking the decomposed uncertainty is the only approach which can satisfy the stealth condition while successfully increasing the AU. The proposed attack on decomposed uncertainty achieves mean AU increases of 0.0286 on MNIST, 0.0853 on CIFAR-10, and 0.1568 on CIFAR-100. On MNIST, the unconstrained AU-only baseline attains a larger raw mean increase (0.0587), but it already violates the stealth constraint on 11.6% of samples; in contrast, the proposed attack preserves feasibility on all samples and still outperforms the feasible subset of that baseline in guaranteed mean damage (0.0286 versus 0.0181 feasible-only mean AU increase). On CIFAR-10 and CIFAR-100, the gap becomes sharper: the largest raw AU gains come from AU-only and entropy-maximizing attacks, but these violate the epistemic constraint on roughly 91–98% of

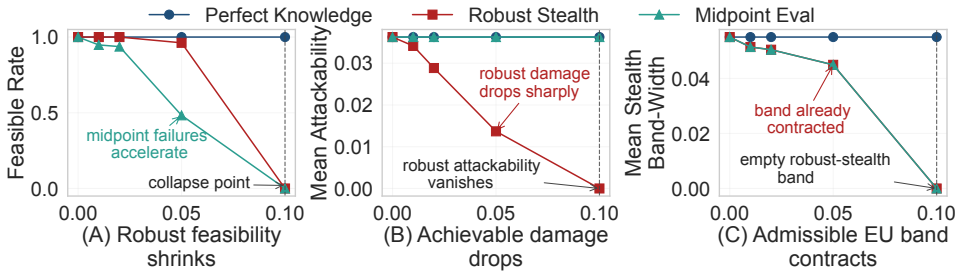

Figure 4: Disclosure-ablation summary on MNIST ResNet56. The x-axis is the attacker's uncertainty radius r around the baseline epistemic level. Panel (A) shows that midpoint-calibrated attacks begin to fail the robust-stealth test well before complete band collapse. Panel (B) shows that the robust attacker achieves progressively less aleatoric damage than the perfect-knowledge attacker as uncertainty about e grows. Panel (C) visualizes the mechanism directly: the admissible epistemic stealth band contracts with r and eventually becomes empty, at which point robust attackability collapses to zero.

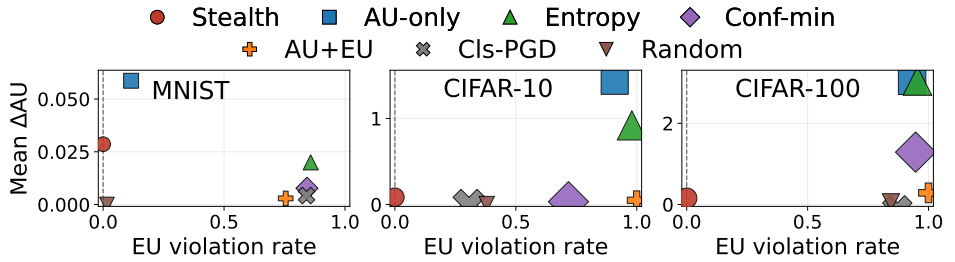

Figure 5: Baseline comparison on ResNet56 across MNIST, CIFAR-10, and CIFAR-100. Each point shows one attack objective, with the x-axis giving the rate of epistemic-stealth violation and the y-axis giving the mean increase in AU. Marker size is proportional to classification accuracy drop. The proposed attack is the only method that stays at zero violation across all three datasets while still producing non-trivial aleatoric damage.

samples and induce large accuracy collapse, making them poor baselines for the stealth-constrained threat model.

Table 3: Compact baseline summary for ResNet56. Each entry reports mean ΔAU / EU-violation rate / accuracy drop. Higher ΔAU is better for the attacker, but lower EU violation is required by the threat model.

Attack objective	MNIST	CIFAR-10	CIFAR-100
Proposed AU+EU stealth	0.0286 / 0.000 / 0.001	0.0853 / 0.000 / 0.147	0.1568 / 0.000 / 0.234
AU-only maximization	0.0587 / 0.116 / 0.010	1.4375 / 0.909 / 0.610	3.0497 / 0.932 / 0.629
Entropy maximization	0.0200 / 0.858 / 0.015	0.9191 / 0.980 / 0.747	3.0427 / 0.954 / 0.704
Classification PGD	0.0042 / 0.841 / 0.073	-0.0093 / 0.307 / 0.950	-0.1438 / 0.871 / 0.768
Random ℓ_∞	0.0000 / 0.016 / 0.000	0.0053 / 0.379 / 0.026	0.0544 / 0.845 / 0.095

Table 3 makes two additional points explicit. First, standard classification attacks are not a good surrogate for uncertainty manipulation as PGD strongly damages accuracy, but it produces negligible or even negative mean AU change on the harder datasets. Second, maximizing scalar uncertainty summaries such as entropy or total uncertainty is not sufficient either, because those objectives typically violate the stealth condition. Overall, these baselines support our claim that uncertainty manipulation is a distinct attack problem – once epistemic stealth is imposed, what matters is how much the DNN is confused without raising any flag to the defenders.

7 Summary of Limitations and Next Steps

In this work, we have studied manipulation of predictive uncertainty through a decomposition-aware lens. We show, both empirically and theoretically, that the maximum aleatoric uncertainty increase is dependent on the precision of information of the epistemic uncertainty disclosed to the attacker. Even then, the samples at intermediate margin region are the ones most vulnerable. Despite our theoretical and empirical findings, our work has several limitations. First, we use the credal set over an ensemble

prediction as the object from which we derive the entropy based uncertainty functionals. While this is consistent with the credal framework, alternate uncertainty decompositions (e.g., Bayesian, Evidential) may deviate from the results presented in this work. Second, our theoretical results are primarily first-order and local, capturing gradient-level geometry under small perturbations. As observed empirically, these approximations are most predictive in simpler regimes and become less sufficient on more complex datasets and architectures. Finally, our study focuses on classification with ensemble-based predictors, and it remains unclear how these phenomena extend to other settings. In future, we plan to extend the analysis to incorporate higher order approximations, diverse settings, and large-scale models (e.g., vision-language models). Beyond analysis, developing defenses, monitoring strategies, and robustness guarantees against such manipulations would be the next immediate step.

Acknowledgments

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A Proof of Theoretical Claims

Theorem A.1 (Iff Characterization: $EU(\mathcal{P}_x) = 0$ Yields a Smaller Maximum Damage Budget)**.** *Fix $\delta > 0$ and assume $EU, AU : \mathfrak{P} \to \overline{\mathbb{R}}$ with $EU(\mathcal{P}) \geq 0$ for all $\mathcal{P} \in \mathfrak{P}$. Let $x_0, x_1 \in \mathcal{X}$ satisfy*

$$EU(\mathcal{P}_{x_0}) = 0, \qquad EU(\mathcal{P}_{x_1}) =: e > 0.$$

Define the following two (nonempty) envelope values,

$$A_0(\delta) := \sup_{\tilde{x} \in \mathcal{X} : EU(\mathcal{P}_{\tilde{x}}) \leq \delta} AU(\mathcal{P}_{\tilde{x}}), \qquad A_1(e, \delta) := \sup_{\tilde{x} \in \mathcal{X} : \max(0, e-\delta) \leq EU(\mathcal{P}_{\tilde{x}}) \leq e+\delta} AU(\mathcal{P}_{\tilde{x}}).$$

Then the following statements are equivalent:

(i) $\Gamma(x_0, \delta) < \Gamma(x_1, \delta)$.

(ii) The strict envelope inequality holds:

$$A_1(e, \delta) - A_0(\delta) > AU(\mathcal{P}_{x_1}) - AU(\mathcal{P}_{x_0}). \tag{7}$$

(iii) There exists $\hat{x} \in \mathcal{X}$ such that $|EU(\mathcal{P}_{\hat{x}}) - e| \leq \delta$ and

$$AU(\mathcal{P}_{\hat{x}}) - AU(\mathcal{P}_{x_1}) > \sup_{\tilde{x} \in \mathcal{X} : EU(\mathcal{P}_{\tilde{x}}) \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_{x_0}) \Big]. \tag{8}$$

In particular, (3) *gives a necessary and sufficient condition for the maximum feasible damage budget (i.e., the largest γ for which the constraints in* (2) *are feasible) to be strictly smaller when $EU(\mathcal{P}_x) = 0$ than when $EU(\mathcal{P}_x) = e > 0$.*

Proof of Theorem 4.3. We first rewrite $\Gamma(x, \delta)$ in a convenient way. By the equivalent definition,

$$\Gamma(x, \delta) = \sup_{\tilde{x} : |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \Big] = \Big[\sup_{\tilde{x} : |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta} AU(\mathcal{P}_{\tilde{x}}) \Big] - AU(\mathcal{P}_x).$$

Now consider x_0 with $EU(\mathcal{P}_{x_0}) = 0$. Since $EU(\cdot) \geq 0$, the stealth constraint becomes

$$|EU(\mathcal{P}_{\tilde{x}}) - 0| \leq \delta \quad \Longleftrightarrow \quad EU(\mathcal{P}_{\tilde{x}}) \leq \delta,$$

so

$$\Gamma(x_0, \delta) = \Big[\sup_{\tilde{x} : EU(\mathcal{P}_{\tilde{x}}) \leq \delta} AU(\mathcal{P}_{\tilde{x}}) \Big] - AU(\mathcal{P}_{x_0}) = A_0(\delta) - AU(\mathcal{P}_{x_0}). \tag{9}$$

Next consider x_1 with $EU(\mathcal{P}_{x_1}) = e > 0$. The stealth constraint is

$$|EU(\mathcal{P}_{\tilde{x}}) - e| \leq \delta \quad \Longleftrightarrow \quad e - \delta \leq EU(\mathcal{P}_{\tilde{x}}) \leq e + \delta.$$

Using again that $EU(\cdot) \geq 0$, we may replace the lower bound $e - \delta$ by $\max(0, e - \delta)$ without changing the feasible set (because $EU(\mathcal{P}_{\tilde{x}}) \geq 0$ automatically). Hence,

$$\Gamma(x_1, \delta) = \Big[\sup_{\tilde{x} : \max(0, e-\delta) \leq EU(\mathcal{P}_{\tilde{x}}) \leq e+\delta} AU(\mathcal{P}_{\tilde{x}}) \Big] - AU(\mathcal{P}_{x_1}) = A_1(e, \delta) - AU(\mathcal{P}_{x_1}). \tag{10}$$

(i) $\Longleftrightarrow$ (ii). Combining (9) and (10), we have

$$\Gamma(x_0, \delta) < \Gamma(x_1, \delta) \quad \Longleftrightarrow \quad A_0(\delta) - AU(\mathcal{P}_{x_0}) < A_1(e, \delta) - AU(\mathcal{P}_{x_1}),$$

which is algebraically equivalent to (3).

(i) $\Longleftrightarrow$ (iii). Let

$$B_0 := \sup_{\tilde{x} : EU(\mathcal{P}_{\tilde{x}}) \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_{x_0}) \Big] = \Gamma(x_0, \delta),$$

$$B_1 := \sup_{\tilde{x} : |EU(\mathcal{P}_{\tilde{x}}) - e| \leq \delta} \Big[AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_{x_1}) \Big] = \Gamma(x_1, \delta).$$

Then (i) is exactly $B_0 < B_1$. If $B_0 < B_1$, by the definition of supremum there exists $\hat{x}$ with $|EU(\mathcal{P}_{\hat{x}}) - e| \leq \delta$ and

$$AU(\mathcal{P}_{\hat{x}}) - AU(\mathcal{P}_{x_1}) > \frac{B_0 + B_1}{2} > B_0,$$

which is precisely (4). Conversely, if (4) holds for some $\hat{x}$, then $B_1 \geq AU(\mathcal{P}_{\hat{x}}) - AU(\mathcal{P}_{x_1}) > B_0$, i.e. $\Gamma(x_1, \delta) > \Gamma(x_0, \delta)$. This proves (i) $\Longleftrightarrow$ (iii). The theorem follows. □

Proposition A.2 (First-Order vulnerability Condition)**.** *Let $a(x) := AU(\mathcal{P}_x)$ and $e(x) := EU(\mathcal{P}_x)$ be continuously differentiable in a neighborhood of x. Define*

$$\Gamma_{\text{lin}}(x, \varepsilon) := \sup_{\|r\|_2 \le \varepsilon,\ \nabla e(x)^\top r = 0} \nabla a(x)^\top r.$$

If $\nabla e(x) \neq 0$, then

$$\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon \left\| \Pi_{\nabla e(x)^\perp} \nabla a(x) \right\|_2,$$

where Π_m denotes orthogonal projection onto the subspace m. If $\nabla e(x) = 0$, then

$$\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon \|\nabla a(x)\|_2.$$

Proof. By first-order Taylor expansion,

$$a(x + r) = a(x) + \nabla a(x)^\top r + o(\|r\|_2), \qquad e(x + r) = e(x) + \nabla e(x)^\top r + o(\|r\|_2)$$

as $\|r\|_2 \to 0$. Under exact first-order epistemic stealth, $\nabla e(x)^\top r = 0$, so feasible directions lie in the hyperplane $\nabla e(x)^\perp$.

Decompose $\nabla a(x) = \Pi_{\nabla e(x)^\perp} \nabla a(x) + \Pi_{\text{span}\{\nabla e(x)\}} \nabla a(x)$. The second term is orthogonal to any feasible r, and Cauchy–Schwarz yields

$$\nabla a(x)^\top r = \left(\Pi_{\nabla e(x)^\perp} \nabla a(x)\right)^\top r \le \left\|\Pi_{\nabla e(x)^\perp} \nabla a(x)\right\|_2 \|r\|_2 \le \varepsilon \left\|\Pi_{\nabla e(x)^\perp} \nabla a(x)\right\|_2,$$

with equality attained at

$$r^\star = \varepsilon \frac{\Pi_{\nabla e(x)^\perp} \nabla a(x)}{\|\Pi_{\nabla e(x)^\perp} \nabla a(x)\|_2}$$

whenever the projected gradient is nonzero. This establishes the first claim.

If $\nabla e(x) = 0$, the stealth constraint is vacuous to first order, and $\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon \|\nabla a(x)\|_2$ follows again by Cauchy–Schwarz. □

Proposition A.3 (Moderate-Margin Peak of Local Aleatoric Sensitivity)**.** *Let $m(x) = z_{(1)}(x) - z_{(2)}(x) > 0$ denote the top-two logit margin at a correctly classified sample x, and define $a(x) := h(\sigma(m(x)))$, where $\sigma(t) = 1/(1 + e^{-t})$ and $h(p) = -p \log p - (1 - p) \log(1 - p)$. Assume m is differentiable near x. Then*

$$\nabla a(x) = -\phi(m(x))\, \nabla m(x), \qquad \phi(t) := \sigma(t)(1 - \sigma(t))\, t,$$

where $\phi(0) = 0$, $\phi(t) \to 0$ as $t \to \infty$, and ϕ attains its unique maximum at the positive solution $t^\star \approx 1.5434$ of $t \tanh(t/2) = 1$.

Hence $\|\nabla a(x)\|_2 = \phi(m(x)) \|\nabla m(x)\|_2$ is maximized, for fixed $\|\nabla m(x)\|_2$, at moderate margin values. If in addition $e(x) := EU(\mathcal{P}_x)$ is differentiable, then under exact first-order epistemic stealth,

$$\Gamma_{\text{lin}}(x, \varepsilon) = \varepsilon\, \phi(m(x))\ \left\|\Pi_{\nabla e(x)^\perp} \nabla m(x)\right\|_2.$$

Proof. Let $p = \sigma(m) = 1/(1 + e^{-m})$ and $a = h(p)$. Direct differentiation gives $dp/dm = \sigma(m)(1 - \sigma(m)) = p(1 - p)$ and $dh/dp = \log((1 - p)/p) = -m$. By the chain rule,

$$\frac{da}{dm} = -m\, p(1 - p),$$

so $|da/dm| = m\, \sigma(m)(1 - \sigma(m)) = \phi(m)$. The multivariate chain rule then yields $\nabla a(x) = \phi(m(x)) \nabla m(x)$, and thus $\|\nabla a(x)\|_2 = \phi(m(x)) \|\nabla m(x)\|_2$.

To analyze ϕ, observe that $\phi(0) = 0$ and $\phi(t) \to 0$ as $t \to \infty$. Differentiating $\phi(t) = t\, \sigma(t)(1 - \sigma(t))$ and setting the derivative to zero leads (after using $1 - 2\sigma(t) = -\tanh(t/2)$) to the equation $t \tanh(t/2) = 1$, whose unique positive solution is $t^\star \approx 1.5434$. This is the global maximizer since ϕ vanishes at the boundaries.

If $e(x)$ is differentiable, the first-order vulnerability condition immediately gives the stated expression for $\Gamma_{\text{lin}}(x, \varepsilon)$ upon substituting $\nabla a(x) = \phi(m(x)) \nabla m(x)$. □

B Empirical predictive object and Operational uncertainty functionals

In the experiments, the predictor object is a CreDRO ensemble with member predictive distributions $\{p_k(\cdot \mid x)\}_{k=1}^{K}$. We associate to x the empirical predictive credal object $\mathcal{P}_x := \mathrm{conv}\{p_1(\cdot \mid x), \ldots, p_K(\cdot \mid x)\}$, that is, the convex hull of the ensemble member predictions. Our attack and analysis use operational uncertainty functionals derived from these member predictions. Let $H(p) := -\sum_y p(y) \log p(y)$ denote Shannon entropy. Then the experimental aleatoric and epistemic quantities are defined as $AU(\mathcal{P}_x) := \min_{1 \le k \le K} H(p_k(\cdot \mid x))$, and $EU(\mathcal{P}_x) := \max_{1 \le k \le K} H(p_k(\cdot \mid x)) - \min_{1 \le k \le K} H(p_k(\cdot \mid x))$. In our implementation, AU is the smallest member entropy and EU is the range of member entropies. These are the quantities used throughout the experiments, including the attack objective, the feasibility constraint, and the reported correlations.

C Implementation Details, Hardware, and Hyperparameter Settings

Hardware Setup All experiments were conducted on NVIDIA A100 GPUs (80GB memory). Depending on the experiment, we used up to 4 GPUs in parallel. The total compute used for the experiments reported in the paper is approximately 1500 GPU hours. Individual training runs typically require several hours to a few days depending on the dataset and model architecture. In addition to the reported experiments, we conducted preliminary and exploratory runs requiring comparable compute, which are not included in the final results.

Training configuration The runtime configuration in the repository uses SGD with momentum 0.9, weight decay 5×10^{-4}, initial learning rate 0.1, and a multistep schedule with drops at epochs 100 and 150 over 200 epochs. CIFAR training uses random crop and horizontal flip; MNIST uses normalization only. The global CreDRO parameter is 0.5, and the five ensemble members span different DRO fractions.

Attack and evaluation protocol For the main attackability experiments, we approximate

$$\Gamma_\epsilon(x, \delta) = \max_{\|\tilde{x} - x\|_\infty \le \epsilon} \big(AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \big)$$

subject to epistemic stealth using a penalty-based projected gradient method over an ℓ_∞ ball. The default reported attack parameters are $\epsilon = 0.12549$, step size 0.007843137, 100 PGD steps, stealth budget $\delta = 0.05$, and penalty weight $\lambda = 5.0$. Operationally, the attack maximizes AU while penalizing violations of $|EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \le \delta$.

The implementation uses up to three adaptive PGD attempts per sample rather than a single run. If an early attempt does not yield a feasible adversarial example, the solver retries with a reduced step size and an increased penalty weight, and it stops once a feasible candidate is found. This provides a lightweight optimization safeguard against declaring low attackability purely because of an unfavorable step size or insufficient penalty strength.

As optimization diagnostics, we report post-attack feasibility under the target EU-stealth constraint together with the empirical distribution of absolute epistemic drift $|EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)|$. In the currently stored runs, the default configuration achieves a feasible rate of 1.0 across the main MNIST, CIFAR-10, and CIFAR-100 dataset–architecture pairs at $\delta = 0.05$, with observed absolute epistemic drifts remaining below the target tolerance. These diagnostics do not replace a full ablation over λ or a projection-based feasibility solver, but they do show that the reported AU gains in the current experiments are not being obtained by overt violations of the intended stealth constraint.

Unless otherwise noted, the reported aggregate analyses are computed on the first 5000 test samples of each dataset–architecture pair in order to keep the complexity manageable while preserving per-sample resolution. For correlation-based claims, we additionally report bootstrapped confidence intervals from 200 resamples.

D Additional Analysis

Optimization Diagnostics From Table 4, we can see that for all our settings, our attack finds the achievable maximum AU within the budgeted EU change.

Table 4: Fraction of attacked samples admitting stealthy attack for optimization diagnostics.

Dataset	Arch	Attempts	Feasible rate	Mean $\lvert\Delta EU\rvert$	Max $\lvert\Delta EU\rvert$
MNIST	ResNet56	3	1.000	0.008	0.050
MNIST	DenseNet	3	1.000	0.009	0.050
MNIST	WideResNet	3	1.000	0.015	0.050
CIFAR-10	ResNet56	3	1.000	0.030	0.050
CIFAR-10	DenseNet	3	1.000	0.031	0.050
CIFAR-10	WideResNet	3	1.000	0.032	0.050
CIFAR-100	ResNet56	3	1.000	0.021	0.050
CIFAR-100	DenseNet	3	1.000	0.022	0.050
CIFAR-100	WideResNet	3	1.000	0.025	0.050

Table 5: Budget-sweep summary on MNIST ResNet56 over the first 1000 test samples. Larger ϵ and δ increase achievable damage, while the projected-gradient and $\phi(m)$ associations remain stable across budgets.

ϵ	δ	Mean Γ	Max Γ	$\rho_s(\Gamma, \lVert\Pi_{\nabla EU^\perp}\nabla AU\rVert_2)$	$\rho_s(\Gamma, \phi(m))$	$\rho_s(\Gamma, m)$
0.062745	0.02	0.0054	0.5762	0.8642	0.6552	-0.6551
0.125490	0.02	0.0173	0.7609	0.7576	0.5462	-0.5462
0.062745	0.05	0.0092	0.7877	0.8767	0.6688	-0.6687
0.125490	0.05	0.0270	1.1045	0.7885	0.5784	-0.5784
0.062745	0.10	0.0110	0.8540	0.8772	0.6696	-0.6695
0.125490	0.10	0.0386	1.3896	0.8017	0.5928	-0.5927

Robustness Across Attack Budgets. To verify that the qualitative picture is not tied to a single attack setting, we run a targeted budget sweep on MNIST ResNet56 over the first 1000 test samples, varying $\delta \in \{0.02, 0.05, 0.10\}$ and $\epsilon \in \{0.062745, 0.125490\}$. Table 5 shows the main pattern compactly. Increasing either budget increases both mean and maximum achievable damage, but the geometric signatures remain stable: the projected-gradient correlation stays uniformly high, the $\phi(m)$ correlation remains positive in every setting, and the raw-margin correlation remains negative. This indicates that the mechanism identified in the main experiments is not an artifact of a single hyperparameter choice.

Bootstrap Confidence Intervals for Correlation Tables The corresponding 95% bootstrap confidence intervals are reported in Appendix Tables .

Table 6: Spearman correlations between vulnerability Γ and first-order attackability descriptors on the representative 1000-sample subsets, with 95% bootstrap confidence intervals from 200 resamples. The table compares the raw aleatoric gradient norm $\lVert\nabla AU\rVert_2$ against its EU-stealth-compatible projected counterpart $\lVert\Pi_{\nabla EU^\perp}\nabla AU\rVert_2$. Small gains in the main text can thus be interpreted relative to their sampling uncertainty.

Dataset	Arch	$\rho_s(\Gamma, \lVert\nabla AU\rVert_2)$	$\rho_s(\Gamma, \lVert\Pi_{\nabla EU^\perp}\nabla AU\rVert_2)$	Gain
MNIST	ResNet56	0.782 [0.752, 0.802]	0.789 [0.758, 0.809]	0.007
	WideResNet	0.593 [0.547, 0.638]	0.650 [0.611, 0.695]	0.056
	DenseNet	0.679 [0.640, 0.712]	0.717 [0.681, 0.753]	0.038
CIFAR-10	ResNet56	0.127 [0.066, 0.189]	0.127 [0.066, 0.189]	-0.000
	WideResNet	0.289 [0.224, 0.364]	0.289 [0.223, 0.364]	-0.000
	DenseNet	0.234 [0.175, 0.304]	0.233 [0.174, 0.302]	-0.001
CIFAR-100	ResNet56	-0.327 [-0.388, -0.252]	-0.325 [-0.388, -0.248]	0.002
	WideResNet	0.026 [-0.050, 0.103]	0.037 [-0.034, 0.115]	0.012
	DenseNet	-0.159 [-0.220, -0.080]	-0.156 [-0.217, -0.079]	0.004

Table 7: Spearman correlations between vulnerability Γ and classifier top-two margin on the representative 1000-sample subsets, with 95% bootstrap confidence intervals from 200 resamples. These intervals quantify the stability of the margin–vulnerability relation across datasets and architectures.

Dataset	Arch	$\rho_s(\Gamma, \text{margin})$ with 95% CI
MNIST	ResNet56	-0.590 [-0.630, -0.540]
	WideResNet	-0.457 [-0.509, -0.406]
	DenseNet	-0.589 [-0.631, -0.545]
CIFAR-10	ResNet56	-0.169 [-0.228, -0.108]
	WideResNet	-0.317 [-0.393, -0.247]
	DenseNet	-0.255 [-0.317, -0.196]
CIFAR-100	ResNet56	0.313 [0.241, 0.368]
	WideResNet	-0.060 [-0.138, 0.019]
	DenseNet	0.140 [0.064, 0.201]

Table 8: Spearman correlations between vulnerability Γ and the margin-modulation factor $\phi(m) = m\,\sigma(m)(1-\sigma(m))$ on the representative 1000-sample subsets, with 95% bootstrap confidence intervals from 200 resamples. We report both the full subset and the correctly classified subset to show how the moderate-margin signal changes after excluding misclassified examples.

Dataset	Arch	Full subset	Correct-only subset
MNIST	ResNet56	0.578 [0.524, 0.616]	0.577 [0.523, 0.620]
	WideResNet	0.335 [0.273, 0.396]	0.337 [0.277, 0.389]
	DenseNet	0.453 [0.397, 0.499]	0.448 [0.385, 0.499]
CIFAR-10	ResNet56	0.202 [0.132, 0.266]	0.249 [0.190, 0.320]
	WideResNet	0.338 [0.273, 0.410]	0.374 [0.307, 0.450]
	DenseNet	–	–
CIFAR-100	ResNet56	-0.159 [-0.213, -0.086]	0.039 [-0.038, 0.109]
	WideResNet	0.105 [0.036, 0.180]	0.220 [0.137, 0.299]
	DenseNet	-0.023 [-0.087, 0.046]	0.132 [0.049, 0.203]

E Relation of Definition 4.2 to feasibility formulations.

The quantity $\Gamma_{\epsilon,\delta}(x)$ admits an equivalent feasibility (dual) interpretation. Instead of maximizing the achievable damage, one may ask whether a target increase γ is feasible under the same constraints, i.e., whether there exists $\tilde{x}$ such that

$$\|\tilde{x} - x\|_\infty \le \epsilon, \quad |EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \le \delta, \quad AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x) \ge \gamma.$$

From this perspective, $\Gamma_\epsilon(x, \delta)$ is the supremum over all feasible damage levels γ. This establishes the equivalence between the maximum-damage formulation used in experiments and the minimum-distance or feasibility-based viewpoints commonly used in adversarial analysis.

F Additional Discussion on Blessing of Apparent Ignorance of EU

We discuss additional interpretation of the results from Theorem 4.3. A possible interpretation of condition (3) is as follows. Following [Walley, 1991], we may posit that in many imprecise probability learning settings EU does not vanish with a finite sample size, so that typically $EU(\mathcal{P}_x) > 0$. In that case, if the defender explicitly computes $EU(\mathcal{P}_x)$ (and this baseline value is accessible to the attacker), then the EU-based stealth constraint in (2) centers the attacker's admissible perturbations around a positive level $e = EU(\mathcal{P}_x)$, potentially granting access to credal sets with substantially higher achievable AU (and therefore damage) than those available under the low-epistemic cap $EU(\mathcal{P}_x) \le \delta$.

Condition (3) states precisely that this "opportunity gap" $A_1(e,\delta) - A_0(\delta)$ dominates any baseline difference $AU(\mathcal{P}_{x_1}) - AU(\mathcal{P}_{x_0})$, so that the attacker has strictly more room to increase AU at x_1 than at x_0. In this sense, *knowledge or disclosure* (or accurate estimation) of the baseline EU can increase vulnerability under EU-based monitoring, while treating the system as "epistemically certain" (or not revealing EU) can shrink the attacker's stealthy action set and reduce the maximal achievable damage.

An Extended Interpretation of Theorem 4.3 We can provide an even easier way to interpret our necessary and sufficient condition for the "blessing of apparent ignorance" phenomenon.

Identifying the Room Available. For a fixed $x \in \mathcal{X}$ and $\delta > 0$, define the maximal feasible increase in AU under EU-based stealth as

$$\Gamma(x,\delta) := \sup_{\tilde{x}\in\mathcal{X}:|EU(\mathcal{P}_{\tilde{x}})-EU(\mathcal{P}_x)|\le\delta} \Big(AU(\mathcal{P}_{\tilde{x}}) - AU(\mathcal{P}_x)\Big).$$

This can be rewritten as a "ceiling minus baseline" identity:

$$\Gamma(x,\delta) = \Big(\sup_{\tilde{x}:|EU(\mathcal{P}_{\tilde{x}})-EU(\mathcal{P}_x)|\le\delta} AU(\mathcal{P}_{\tilde{x}})\Big) - AU(\mathcal{P}_x).$$

Hence $\Gamma(x,\delta)$ measures the *room* to increase AU while remaining stealthy.

Two Different Ceilings. Assume $EU(\mathcal{P}) \ge 0$ for all predictive credal sets $\mathcal{P}$. If $EU(\mathcal{P}_{x_0}) = 0$, then the stealth constraint reduces to

$$|EU(\mathcal{P}_{\tilde{x}}) - 0| \le \delta \quad\Longleftrightarrow\quad EU(\mathcal{P}_{\tilde{x}}) \le \delta,$$

so the best attainable AU under stealth is

$$A_0(\delta) := \sup_{\tilde{x}:EU(\mathcal{P}_{\tilde{x}})\le\delta} AU(\mathcal{P}_{\tilde{x}}), \qquad \text{and thus} \qquad \Gamma(x_0,\delta) = A_0(\delta) - AU(\mathcal{P}_{x_0}).$$

If instead $EU(\mathcal{P}_{x_1}) = e > 0$, then stealth requires

$$|EU(\mathcal{P}_{\tilde{x}}) - e| \le \delta \quad\Longleftrightarrow\quad e-\delta \le EU(\mathcal{P}_{\tilde{x}}) \le e+\delta,$$

and the best attainable AU under stealth is

$$A_1(e,\delta) := \sup_{\tilde{x}:e-\delta\le EU(\mathcal{P}_{\tilde{x}})\le e+\delta} AU(\mathcal{P}_{\tilde{x}}), \qquad \text{and thus} \qquad \Gamma(x_1,\delta) = A_1(e,\delta) - AU(\mathcal{P}_{x_1}).$$

Opportunity Gap. The difference

$$A_1(e,\delta) - A_0(\delta)$$

is the *opportunity gap*: it compares the maximal AU attainable in the EU-band $[e-\delta, e+\delta]$ (relevant when the baseline EU is $e > 0$) to the maximal AU attainable under the low-epistemic cap $EU \le \delta$ (relevant when the baseline EU is 0).

Dominating Baseline Differences. Saying that the attacker has strictly more room to increase AU at x_1 than at x_0 means

$$\Gamma(x_1,\delta) > \Gamma(x_0,\delta).$$

Substituting the expressions above yields

$$A_1(e,\delta) - AU(\mathcal{P}_{x_1}) > A_0(\delta) - AU(\mathcal{P}_{x_0}),$$

which is equivalent (by rearranging terms) to the envelope condition

$$A_1(e,\delta) - A_0(\delta) > AU(\mathcal{P}_{x_1}) - AU(\mathcal{P}_{x_0}).$$

Thus, when the opportunity gap $A_1(e,\delta) - A_0(\delta)$ dominates the baseline difference $AU(\mathcal{P}_{x_1}) - AU(\mathcal{P}_{x_0})$, the attacker has strictly more room to increase AU under EU-based stealth monitoring at x_1 than at x_0.

A simplified exampled: We show the blessing of (apparent) ignorance of EU with an example in binary classification Let $\mathcal{Y} = \{0,1\}$ and identify each $P \in \Delta_{\mathcal{Y}}$ with a Bernoulli distribution $\mathrm{Bern}(p)$, $p \in [0,1]$. Consider a parametric family of credal sets given by intervals of Bernoulli parameters:

$$\mathcal{P}(m,u) := \Big\{\mathrm{Bern}(p) : p \in \big[m - \tfrac{u}{2}, m + \tfrac{u}{2}\big] \cap [0,1]\Big\}, \qquad m \in [0,1], u \in [0,1].$$

Define EU as *imprecision width*

$$EU(\mathcal{P}(m,u)) := u,$$

and AU as *lower entropy*

$$AU(\mathcal{P}(m,u)) := \inf_{p\in[m-u/2,m+u/2]\cap[0,1]} h(p),$$

where $h(p) := -p\log_2 p - (1-p)\log_2(1-p)$ is the binary Shannon entropy.

We now impose a restriction on the set of reachable credal sets $\mathfrak{P}$ (as the input varies), capturing the idea that low EU is encountered only in "easy" regions with strongly biased predictions, whereas moderate EU is encountered near decision boundaries where predictions are closer to $1/2$. Concretely, fix $\delta = 0.1$ and $e = 0.4$ and assume:

- For all reachable credal sets with $EU = u \in [0,\delta]$, the center is $m_0 = 0.05$, i.e. $\mathcal{P}_{\tilde{x}} = \mathcal{P}(0.05, u)$.
- For all reachable credal sets with $EU = u \in [e-\delta, e+\delta] = [0.3, 0.5]$, the center is $m_1 = 0.5$, i.e. $\mathcal{P}_{\tilde{x}} = \mathcal{P}(0.5, u)$.

Choose baseline points x_0, x_1 such that

$$\mathcal{P}_{x_0} = \mathcal{P}(0.05, 0) \quad \big(EU(\mathcal{P}_{x_0}) = 0\big), \qquad \mathcal{P}_{x_1} = \mathcal{P}(0.5, 0.4) \quad \big(EU(\mathcal{P}_{x_1}) = e\big).$$

We compute the envelopes in (3). For $u \in [0, 0.1]$ centered at 0.05, the lower entropy equals $h(0.05 - u/2)$, which decreases with u; hence the maximum over this family is attained at $u = 0$, yielding

$$A_0(\delta) = \sup_{u\in[0,\delta]} AU(\mathcal{P}(0.05,u)) = AU(\mathcal{P}(0.05,0)) = h(0.05) \approx 0.2864.$$

For the band $u \in [0.3, 0.5]$ centered at 0.5, the lower entropy equals $h(0.5 - u/2)$, which is larger when u is smaller; thus

$$A_1(e,\delta) = \sup_{u\in[0.3,0.5]} AU(\mathcal{P}(0.5,u)) = AU(\mathcal{P}(0.5,0.3)) = h(0.35) \approx 0.9341.$$

Moreover,

$$AU(\mathcal{P}_{x_0}) = h(0.05) \approx 0.2864, \qquad AU(\mathcal{P}_{x_1}) = AU(\mathcal{P}(0.5,0.4)) = h(0.30) \approx 0.8813.$$

Therefore,

$$A_1(e,\delta) - A_0(\delta) \approx 0.9341 - 0.2864 = 0.6477 \quad > \quad 0.8813 - 0.2864 = 0.5949 = AU(\mathcal{P}_{x_1}) - AU(\mathcal{P}_{x_0}),$$

and condition (3) is satisfied.

The above construction illustrates the same structural mechanism behind (3):

- In the low-epistemic region $\{EU(\mathcal{P}) \le \delta\}$, the reachable credal sets have comparatively small maximal AU, so $A_0(\delta)$ is small.
- In the band $\{EU(\mathcal{P}) \in [e-\delta, e+\delta]\}$ around a positive baseline $e > 0$, the reachable credal sets include substantially larger AU, so $A_1(e,\delta)$ is much larger.

When this "opportunity gap" $A_1(e,\delta) - A_0(\delta)$ dominates any baseline difference in $AU(\mathcal{P}_x)$, the attacker has strictly more room to increase AU under EU-based stealth monitoring.

G Baseline Attack Details

The baseline-comparison experiment in Section 5 uses the same perturbation budget, step size, and number of PGD iterations as the main proposed attack, but replaces the optimization objective. For an ensemble with member predictions $\{p_k(\cdot \mid x)\}_{k=1}^K$, let $\bar{p}(\cdot \mid x) = \frac{1}{K}\sum_{k=1}^K p_k(\cdot \mid x)$ denote the ensemble-mean predictive distribution, let $H(\bar{p})$ denote predictive entropy, and let $\hat{y}(x) = \arg\max_c \bar{p}_c(\cdot \mid x)$.

The compared attacks are:

Table 9: Appendix baseline summary for ResNet56. Each entry reports mean ΔAU / mean $|\Delta EU|$ / EU-violation rate / label-flip rate / accuracy drop.

Attack objective	MNIST	CIFAR-10	CIFAR-100
Proposed AU+EU stealth	0.0286 / 0.0081 / 0.000 / 0.002 / 0.001	0.0853 / 0.0300 / 0.000 / 0.179 / 0.147	0.1568 / 0.0211 / 0.000 / 0.329 / 0.234
Classification PGD	0.0042 / 0.4110 / 0.841 / 0.073 / 0.073	-0.0093 / 0.1532 / 0.307 / 0.953 / 0.950	-0.1438 / 0.7000 / 0.871 / 0.846 / 0.768
Entropy maximization	0.0200 / 0.7498 / 0.858 / 0.018 / 0.015	0.9191 / 0.6494 / 0.980 / 0.795 / 0.747	3.0427 / 0.6403 / 0.954 / 0.935 / 0.704
Confidence minimization	0.0077 / 0.4283 / 0.843 / 0.040 / 0.035	0.0330 / 0.4585 / 0.718 / 0.745 / 0.695	1.2905 / 0.5771 / 0.947 / 0.929 / 0.700
Total uncertainty maximization	0.0029 / 0.6297 / 0.755 / 0.001 / 0.000	0.0494 / 2.0063 / 1.000 / 0.204 / 0.163	0.2878 / 3.1235 / 1.000 / 0.380 / 0.206
AU-only maximization	0.0587 / 0.0401 / 0.116 / 0.011 / 0.010	1.4375 / 0.2286 / 0.909 / 0.659 / 0.610	3.0497 / 0.6384 / 0.932 / 0.854 / 0.629
Random ℓ_∞	0.0000 / 0.0032 / 0.016 / 0.000 / 0.000	0.0053 / 0.1378 / 0.379 / 0.051 / 0.026	0.0544 / 0.4399 / 0.845 / 0.231 / 0.095

- **Proposed AU+EU stealth**: projected-gradient attack maximizing $AU(\mathcal{P}_{\tilde{x}})$ with a penalty for violating $|EU(\mathcal{P}_{\tilde{x}}) - EU(\mathcal{P}_x)| \leq \delta$.
- **Classification PGD**: untargeted PGD maximizing the per-sample negative log-likelihood $-\log \bar{p}_y(\cdot \mid \tilde{x})$ of the true label.
- **Entropy maximization**: PGD maximizing $H(\bar{p}(\cdot \mid \tilde{x}))$.
- **Confidence minimization**: PGD minimizing $\max_c \bar{p}_c(\cdot \mid \tilde{x})$, equivalently maximizing its negative.
- **Total uncertainty maximization**: PGD maximizing $AU(\mathcal{P}_{\tilde{x}}) + EU(\mathcal{P}_{\tilde{x}})$. In the current repository implementation, this equals the maximum member entropy.
- **AU-only maximization**: PGD maximizing $AU(\mathcal{P}_{\tilde{x}})$ without any epistemic-stealth penalty.
- **Random ℓ_∞**: non-adaptive random perturbation sampled uniformly from the same ℓ_∞ ball.

For all baselines, clean and adversarial labels are computed from the ensemble-mean prediction. The uncertainty decomposition used in evaluation matches the repository implementation: AU is the minimum member entropy, and EU is the difference between maximum and minimum member entropy across ensemble members. The summary statistics reported in the main text and below are computed over 5000 test samples for ResNet56 on MNIST, CIFAR-10, and CIFAR-100. We report mean ΔAU, mean absolute ΔEU, epistemic-constraint violation rate, label-flip rate, and accuracy drop.

Table 9 shows that the proposed method is the only attack that consistently combines non-trivial aleatoric damage with negligible epistemic drift. The remaining objectives each fail in a different way: classification PGD mostly damages accuracy rather than AU, entropy/confidence/total-uncertainty baselines substantially distort EU, and the AU-only baseline attains high raw AU gain at the cost of widespread stealth violations on CIFAR-10 and CIFAR-100. This is why the main text evaluates the baselines through the joint lens of AU gain and EU-feasibility rather than raw uncertainty distortion alone.

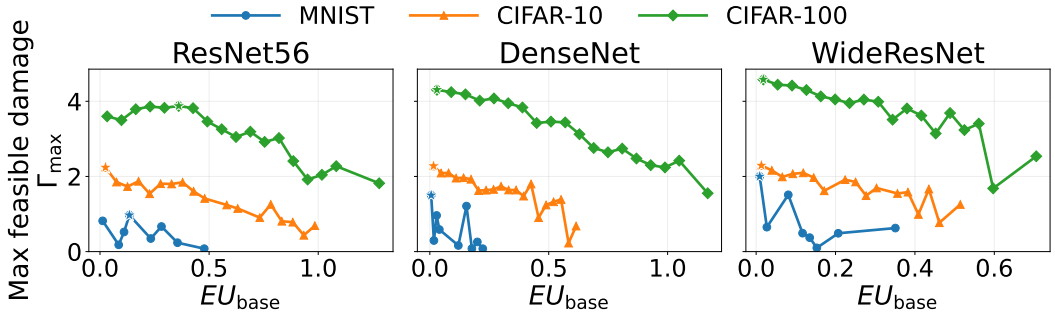

Figure 6: Maximum feasible aleatoric damage as a function of baseline epistemic uncertainty under the standard deep-ensemble uncertainty decomposition. For each dataset and architecture, we bin samples by EU_{base} and report the upper envelope of feasible damage within each bin. The peak damage typically occurs away from both extremes, corroborating the intermediate-EU vulnerability pattern under the standard expected-entropy / mutual-information decomposition.

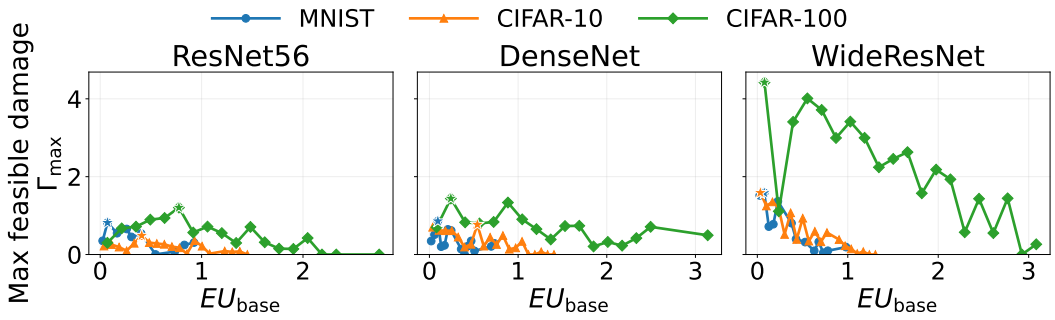

Figure 7: Maximum feasible aleatoric damage as a function of baseline epistemic uncertainty for the label-preserving attack variant. Here the adversarial example is constrained to keep the ensemble prediction unchanged, $\hat{y}(\tilde{x}) = \hat{y}(x)$. The resulting curves remain non-monotone across datasets and architectures, indicating that the intermediate-EU damage peak is not an artifact of label changes.

H Non-monotonic attackability under attack variants

To verify that the main non-monotonicity phenomenon is not specific to our original credal decomposition or to attacks that allow prediction changes, we repeated the damage-versus-baseline-EU analysis for two additional attack settings. First, we reran the proposed attack using the standard deep-ensemble uncertainty decomposition, where AU is the ensemble-averaged entropy and EU is the mutual-information term. Second, we evaluated a label-preserving variant of the proposed attack, which enforces $\hat{y}(\tilde{x}) = \hat{y}(x)$ in addition to the ℓ_∞ and epistemic-stealth constraints. For each dataset and architecture, we binned test samples by baseline epistemic uncertainty EU_{base} and plotted the upper envelope of feasible aleatoric damage, i.e., the maximum observed ΔAU within each bin among attack outcomes satisfying the prescribed stealth constraint. Figures 6 and 7 show that the same qualitative pattern persists in both settings: the largest feasible damage typically does not occur at the lowest or highest baseline epistemic levels, but rather at intermediate values of EU_{base}.

I Limitations

Our analysis relies on several assumptions that may limit its generality. First, the empirical study instantiates uncertainty using entropy-based functionals over ensemble predictions, where aleatoric uncertainty is defined as the minimum member entropy and epistemic uncertainty as the entropy range. While this choice is consistent with the credal framework, alternative uncertainty decompositions (e.g., Bayesian posterior variance or mutual information) may exhibit different behavior under attack. Second, the theoretical analysis is primarily first-order and local, characterizing vulnerability through gradient geometry under small perturbations. As observed empirically, this approximation is strongly predictive in simpler settings but becomes less sufficient on more complex datasets, suggesting that higher-order effects, optimization dynamics, and representation heterogeneity play an important role in harder regimes. Finally, our experiments focus on classification tasks with ensemble-based predictors. Extending the analysis to other settings, such as large-scale foundation models, structured prediction tasks, or non-ensemble uncertainty estimators, remains an open direction.

J Broader Impact

This work studies adversarial manipulation of predictive uncertainty in machine learning systems. On the positive side, our findings can help improve the reliability and safety of uncertainty-aware models by identifying previously overlooked vulnerabilities in uncertainty decomposition. In safety-critical applications such as autonomous driving, medical decision support, and risk-sensitive classification, understanding how uncertainty signals can be manipulated is essential for designing robust monitoring and defense mechanisms.

At the same time, the proposed attack formulation may introduce potential risks if misused. In particular, adversaries could exploit these insights to manipulate uncertainty estimates without triggering standard epistemic alarms, potentially causing systems to misinterpret adversarial inputs

as naturally ambiguous cases. Such vulnerabilities could undermine trust in uncertainty-aware decision-making systems.

We emphasize that this work is intended as a diagnostic and analytical contribution rather than a deployment tool. We hope that exposing these vulnerabilities will encourage the development of more robust uncertainty estimation methods, improved monitoring strategies, and defenses against uncertainty-targeted attacks. Future work should investigate safeguards, detection mechanisms, and robustness guarantees for uncertainty-aware systems.

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